

Axion detection by ring lasers*

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February 1, 2008

Abstract

A ring-laser experiment, similar to the Canterbury ring laser, to detect axion- and QED-induced vacuum birefringence is proposed. It uses a slowly modulated magnetic field and a novel polarization geometry. Both axion coupling and vacuum birefringence would modulate the Sagnac beat frequency. A null result could place sensitive bounds on the axion mass and on two-photon coupling.

The axion was originally introduced [1] to explain the absence of CP violation in the strong interaction. Axions also arise in supersymmetric and superstring theories and are candidates for dark matter in the universe. Laboratory experiments and a host of astrophysical arguments [2] constrain the axion mass to between 10^{-6} eV and 10^{-3} eV. A number of optical experiments have searched for such axions which couple to two photons either directly or via “triangle diagrams” of charged particles. Sikivie [3] suggested that relic axions could be detected through their conversion to photons in a resonant cavity bathed in a strong magnetic field. Experiments of this sort, which rely on the assumption that axions form a significant component of the galactic halo, have yet to confirm the presence of axions [4]. Attempts to produce axions and detect their

*hep-ph/9502256, submitted to Phys. Lett. B

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reconversion into photons, the so-called “shining-light-through-walls” experiment, [5] have, so far, given null results [6]. Maiani et al. [7, 8] have suggested that axions induce small perturbations in the polarization state of a laser beam propagating in a magnetic field. The absence of optical rotation and ellipticity in the transmitted beam has been used to place limits on the axion mass and coupling to two photons [6, 9]. We adapt the ideas of Maiani et al. to propose and examine the feasibility for a ring-laser experiment suitable for detecting axion- and QED-induced vacuum birefringence. We suggest that a ring-laser experiment could play a significant role in the search for axions.

Ring lasers have primarily been used to detect absolute rotation [10]. A ring laser is, however, much more than a gyroscope [11]. A rotational frequency splitting results from the nonreciprocal change ΔL in the optical path lengths for the counterpropagating beams. Viewed from the rotating system, this may be regarded as the consequence of a difference between the effective refractive indices for the two beams. Conversely, any birefringence resulting in a difference Δn in the refractive indices will induce a beat frequency,

$$\Delta f = \frac{f \Delta L}{L} = \frac{f \Delta n l}{L}, \quad (1)$$

where l is the length of the cavity over which the birefringence occurs, L is the ring perimeter and f the optical frequency. In this sense, a ring laser acts as a sensitive differential refractometer [12]. More generally, it has recently been shown that the beat frequency in a ring laser originates in either time-reversal or parity violating effects, depending on the polarization geometry [13]. The He Ne ring laser system installed at Cashmere, Christchurch, New Zealand (henceforth called the Canterbury ring laser), has attained [14] a frequency resolution $\delta f \sim 140$ nHz, or 1 part in 3×10^{21} of the optical frequency $f = 474$ THz. From Eq. (1), this in turn implies a measurement capability $\Delta n = \Delta L/l \sim 10^{-20}$, for a nonreciprocal contribution to the refractive index in a sample of length $l = 10$ cm with $L = 3.4771$ m. At this level of precision, a ring-laser detection of axions and even QED photon-photon coupling becomes feasible. The potential for a precision ring-laser to place sensitive bounds on field-induced birefringence of the vacuum has been recognized [11, 15]. However, the specific details for such experiments have not previously been discussed.

In a square planar-ring, the out-of-plane (s) component of electric field is usually excited rather than the in-plane (p) component since, according to the Fresnel reflection coefficients, the p reflection is inevitably more lossy for oblique incidence. The proposed polarization geometry is illustrated in Fig. 1. Two Faraday cells, whose optic axes are diametrically opposed in the direction of beam propagation, are placed at each end of one of the legs of the ring. The polarization vector $\vec{s} = (1, 0)$ for the incident beam is rotated successively into $(a, b) = R(\theta)(1, 0)$ and $R(-\theta)(a, b) = \vec{s}$ by the Faraday cell at each end of the leg. Accordingly, s polarization is maintained in the ring as a whole, thus ensuring high mirror reflectivity. Within the leg, the counterpropagating beams will have

mutually orthogonal polarizations for a Faraday rotation angle $\theta = 45^\circ$. Such a polarization geometry could feasibly be engaged by a pair of terbium-gallium-garnet (TGG) Faraday rotators (Litton-Airtron Industries), cut at Brewster's angle, which have at least 99.9% transmission at 633.0 nm. Such a total loss, of the order of 1000 ppm, would degrade the cavity quality factor implied above by a factor of up to 100, and requires fuller design work. We propose at least the use of Brewster-cut end faces to help ensure that the fraction of these losses contributing to backscatter and so frequency pulling remains at an acceptable level (say ≤ 50 ppm).

The coupling of axions to two photons via the triangle anomaly is described by the effective lagrangian density

$$\mathcal{L} = \frac{1}{M} (\vec{E} \cdot \vec{B}_{\text{ext}}) a, \quad (2)$$

where \vec{E} is the electric field vector of the laser beam propagating through an applied (static) magnetic field \vec{B}_{ext} and a is the pseudoscalar axion field. The inverse coupling $M = g_{\gamma\gamma a}^{-1}$ has dimensions of energy and a natural interpretation as a symmetry-breaking scale. If the axion mass m_a is less than the photon energy ω , axions can be produced through the Primakoff effect [16] according to the graph in Fig. 2 (a). In view of Eq. (2), the component of \vec{E} parallel to \vec{B}_{ext} is attenuated while the orthogonal component is unaffected. Production of virtual axions is shown in Fig. 2 (b), whereby the parallel component of the incident laser beam oscillates to a massive axion for part of its travel, and so is retarded with respect to the orthogonal component. This phase shift would be converted to a frequency shift in an active ring laser, as the corresponding wavelength stretches or shrinks to accommodate the new round-trip optical path length. This effect can take place even if $m_a > \omega$.

When the mixing of the photon and axion fields is weak [8], the phase shift and attenuation of \vec{E}_{\parallel} are

$$\delta(l) = 2 \left(\frac{\omega B_{\text{ext}}}{M m_a^2} \right)^2 \sin^2 \left(\frac{m_a^2 l}{4\omega} \right), \quad (3)$$

$$\phi_a(l) = \left(\frac{\omega B_{\text{ext}}}{M m_a^2} \right)^2 \left(\frac{m_a^2 l}{2\omega} - \sin \frac{m_a^2 l}{2\omega} \right), \quad (4)$$

where l is the length of the magnetic field region. Up to a factor of $L = Nl$, these results agree with Eqs. (42) and (43) of Ref. [8]. This factor arises in multipass cavity ellipsometry [6, 9] where the laser beam is multiply reflected back and forth N times through the magnetic field so as to accumulate a larger effect. The measured rotation, $\epsilon = \delta/2$, and ellipticity, $\psi = \phi/2$, are cumulative upon reflection so that for N reflections, the values of ϵ and ψ given by Eqs. (3) and (4) are increased by a factor of N . In an active ring-laser, however, the frequency shift is determined solely by the nonreciprocal change in the optical

path length for one complete circuit of the ring. In this sense, the proposed ring-laser experiment is a single-pass experiment with $N = 1$ and $L = l$.

For photon energies $\omega < 2m_e$, the birefringence due to the QED vacuum-polarization graph of Fig. 2 (c) is [17]:

$$\phi_{\text{QED}}(l) = \frac{2\alpha^2 B_{\text{ext}}^2}{15m_e^4} \omega l. \quad (5)$$

where m_e is the electron mass and $\alpha = e^2/4\pi \approx 1/137$ is the fine structure constant. Vacuum birefringence is particularly interesting as a test of higher-order QED perturbation theory which has not, as yet, been measured directly with real incident photons [18].

If \vec{B}_{ext} is modulated at low frequency, then the selective attenuation and retardation of one of the ring-laser beams will induce AM and FM sideband peaks to the earth's rotation-induced Sagnac beat frequency, with peak amplitude and phase deviations δ and ϕ , respectively. Analyzing the amplitude, phase and harmonic content of the sideband spectrum should thus be sufficient to determine the mass, coupling and parity of the axion. The details of sideband analysis in a precision ring laser has been discussed elsewhere [14]. For FM at a frequency f_m , we may write the instantaneous ring-laser beat frequency as

$$f = f_c - \Delta f \sin \omega_m t,$$

where Δf is the peak frequency deviation from the Sagnac carrier frequency f_c . The resultant FM signal can be expanded as

$$\begin{aligned} V(t) \Big|_{\text{FM}} &= \Re \{ V_c \exp i(\omega_c t + \beta \cos \omega_m t) \} \\ &= \Re \left\{ V_c (\exp i\omega_c t) \left(J_0(\beta) + 2 \sum_{n=1}^{\infty} i^n J_n(\beta) \cos(n\omega_m t) \right) \right\}, \end{aligned}$$

where the modulation index $\beta = \Delta f/f_m$ is the amplitude (in radians) of the phase excursion induced by the frequency modulation and

$$J_n(\beta) = \left(\frac{\beta}{2} \right)^n \sum_{m=0}^{\infty} (-1)^m \frac{(\beta/2)^{2m}}{m!(m+n)!} \quad (6)$$

is the n th-order Bessel function. This result tells us that with single-frequency dithering, there will be an infinite series of sideband satellites above and below the earth line, separated in frequency precisely by f_m , with amplitudes controlled by $J_n(\beta)$. When $\beta \ll 1$, Eq. (6) may be expanded as

$$J_0(\beta) \approx 1 - \frac{\beta^2}{4}, \quad J_1(\beta) \approx \frac{\beta}{16}(8 - \beta^2) \quad \text{and} \quad J_2(\beta) \approx \frac{\beta^2}{96}(12 - \beta^2).$$

Interpreting the ring-laser detection voltage as quantifying a light beam intensity rather than an amplitude, the heights of the lowest-order sidebands are N decibels below the carrier, where

$$N = 10 \log_{10} \left(\frac{J_0(\beta)}{J_1(\beta)} \right) \approx 10 \log_{10} \left(\frac{2}{\beta} \left(1 - \frac{1}{8} \beta^2 \right) \right) \approx 3 - 10 \log_{10} \beta. \quad (7)$$

The carrier amplitude itself is reduced by $-10 \log_{10} J_0(\beta) \approx \beta^2$ dB. Note that contrary to standard engineering practice, we follow Ref. [14] and define the decibel as $10 \log_{10} V$, rather than $20 \log_{10} V$, since the photomultiplier signal V is already proportional to the light intensity, rather than its amplitude. From Eq. (7), a sideband which is $10n$ dB below the carrier translates into a modulation index $\beta = 2 \times 10^{-n}$ and so to a peak frequency deviation $\Delta f = 2 \times 10^{-n} f_m$.

Fig. 3 plots the signal-to-noise (S/N) ratios thus required to detect a given frequency shift Δf for μHz , mHz and Hz modulation frequencies. The shaded region indicates the present sensitivity of the Canterbury ring laser: for a modulation frequency of 1 Hz, a peak frequency shift of 1 μHz would yield sidebands (displaced 1 Hz either side of the Sagnac carrier) which were resolvable above the noise. From Eq. (1), this translates into a sensitivity for detecting a non-reciprocal contribution $\Delta n = 7.3 \times 10^{-20}$ to the refractive index in a sample of length $l = 10$ cm. This in turn implies a measurement capability

$$\Delta \phi = \frac{2\pi f \Delta n l}{c} = 7.3 \times 10^{-14} \quad (8)$$

for the axion-induced phase shift between the counterpropagating beams.

Conversely, an observed *absence* of AM and FM sidebands would, by virtue of Eqs. (3) and (4), place upper bounds on the axion-photon coupling $g_{\gamma\gamma a} = M^{-1}$. These bounds are illustrated in Fig. 4 as a function of axion mass, assuming a ring-laser sensitivity as given in Eq. (8), for a 1 T magnetic field applied across a 10 cm section of the beam paths. Various experimental and theoretical bounds on the axion have been summarized in Ref. [6] and are also shown in Fig. 4. For $m_a < 10^{-3}$ eV, we see that the optical-rotation bound of the Brookhaven-Fermilab-Rochester (BFR) multipass cavity experiment is an order-of-magnitude better than that which could be achieved by a ring-laser experiment with μHz frequency resolution. This is despite the fact that the ring-laser experiment could detect phase shifts $\phi \sim 10^{-13}$ rad, some 10^4 -times better than the corresponding measurements made in the BFR experiment. This is because the magnitude of the induced effects depend critically upon the strength of the magnetic field and the optical path length $L = Nl$ through the field region. However, for relatively large axion masses, $m_a > 2 \times 10^{-3}$ eV, the ring laser could surpasses the corresponding BFR ellipticity bound. This is because the ellipticity measurements were three times less sensitive than the optical rotation measurements due to larger systematic effects and also a more pronounced effect of random motion of the beam on the ellipsometer optics [6].

With improved sensitivity, the ring laser could probe higher effective energies, thus placing tighter bounds on the axion mass and coupling to two photons. In order to detect QED vacuum birefringence, Eq. (5) implies that a beat frequency resolution of 54 pHz would be required. Fig. 3 shows that for modulation of \vec{B}_{ext} at 1 Hz, the QED-induced sidebands would lie 110 dB below the Sagnac carrier. Although the present signal-to-noise ratio is approximately 60–90 dB on a hertz-wide scale, a synchronous filter could feasibly extract such a signal from the noise. We have not explored the limits of phase-sensitive or similar such AC methods of detection. Indeed, the possibility of a ring-laser detection of QED vacuum birefringence provides strong incentive for further research in this direction.

To summarize, astrophysical and laboratory constraints leave but one window open for the axion mass: 10^{-6} eV to 10^{-3} eV. The proposed ring-laser experiment could search for axions in this mass range which couple to two photons up to an effective energy of 10^5 GeV, assuming microhertz resolution of the ring-laser beat frequency. The BFR experiment [6], however, has already ruled out the existence of such axions with couplings below 3×10^6 GeV. Moreover, calculations based upon stellar evolution [2] limit $M > 3 \times 10^9$ GeV for axions of mass 10^{-6} eV to 10^{-3} eV. Nevertheless, a ring-laser experiment could still provide independent insight into the question of axion parameters.

We emphasize that the above estimates have been based upon the level of precision (1 μ Hz) achieved by the Canterbury ring laser in early 1994, and which has already been exceeded by a factor 6. With improved frequency resolution, assisted by phase-sensitive methods of detection, the bounds placed by a ring-laser experiment could surpass those bounds already placed by the BFR experiment. Construction of a second-generation ring-laser at Canterbury has been initiated, which should afford up to a 100-fold improvement in resolution [10]. At this level of precision, we believe that a ring-laser experiment to place sensitive new bounds on the axion, by searching for axions in the mass range 10^{-6} eV to 10^{-3} eV which couple to two photons up to an effective energy of 2×10^7 GeV, would be a realistic goal. This goal would be further motivated by the possibility of a ring-laser detection of QED vacuum birefringence. We suggest that the time is opportune to commence an experiment on the above lines in parallel with the theoretical investigation of external-field-induced effects in ring lasers.

Acknowledgements

LC acknowledges partial support from a University of Canterbury Masters' Scholarship, and advice from Scott Griffen, Litton-Airtron Industries re specifications of TGG Faraday rotators.

References

- [1] R.D. Peccei and H.R. Quinn, Phys. Rev. Lett. 38 (1977) 1440;
R.D. Peccei and H.R. Quinn, Phys. Rev. D 16 (1977) 1791;
S. Weinberg, Phys. Rev. Lett. 40 (1978) 223;
F. Wilczek, Phys. Rev. Lett. 40 (1978) 279.
- [2] M.S. Turner, Phys. Rep. 197 (1990) 67;
G.G. Raffelt, Phys. Rep. 198 (1990) 1;
M.A. Bershadsky, M.T. Ressell and M.S. Turner, Phys. Rev. Lett. 66 (1991) 1398.
- [3] P. Sikivie Phys. Rev. Lett. 51 (1983) 1415, Erratum 52 (1984) 695;
P. Sikivie Phys. Rev. D 32 (1985) 2988.
- [4] S. DePanfilis, A.C. Melissinos, B.E. Moskowitz, J.T. Rogers, Y.K. Semertzidis W.U. Wuensch, H.J. Halama, A.G. Prodell, W.B. Fowler and F.A. Nezrick, Phys. Rev. Lett. 59 (1987) 839;
W.U. Wuensch, S. DePanfilis-Wuensch, Y.K. Semertzidis, J.T. Rogers, A.C. Melissinos, H.J. Halama, B.E. Moskowitz, A.G. Prodell, W.B. Fowler and F.A. Nezrick Phys. Rev. D 40 (1989) 3153;
C. Hagmann, P. Sikivie, N.S. Sullivan and D.B. Tanner, Phys. Rev. D 42 (1990) 1297;
D.M. Lazarus, G.C. Smith, R. Cameron, A.C. Melissinos, G. Ruoso, Y.K. Semertzidis and F.A. Nezrick, Phys. Rev. Lett. 69 (1992) 2333.
- [5] A.A. Ansel'm, Sov. J. Nucl. Phys. 42 (1985) 936;
K. van Bibber, N.R. Dagdeviren, S.E. Koonin, A.K. Kerman and H.N. Nelson Phys. Rev. Lett. 59 (1987) 759.
- [6] R. Cameron, G. Cantatore, A.C. Melissinos, G. Ruoso, Y. Semertzidis, H.J. Halama, D.M. Lazarus, A.G. Prodell, F. Nezrick, C. Rizzo and E. Zavattini, Phys. Rev. D 47 (1993) 3707.
- [7] L. Maiani, R. Petronzio and E. Zavattini, Phys. Lett. B 175 (1986) 359.
- [8] G. Raffelt and L. Stodolsky, Phys. Rev. D 37 (1988) 1237.
- [9] Y. Semertzidis, R. Cameron, G. Cantatore, A.C. Melissinos, J. Rogers, H.J. Halama, A.G. Prodell, F. Nezrick, C. Rizzo and E. Zavattini, Phys. Rev. Lett. 64 (1990) 2988.
- [10] R. Anderson, H.R. Bilger and G.E. Stedman, Am. J. Phys. 62 (1994) 975.
- [11] G.E. Stedman, H.R. Bilger, Z. Li, M.P. Poulton, C.H. Rowe, I. Vetharaniam and P.V. Wells, Aust. J. Phys. 46 (1993) 87.

- [12] H.R. Bilger, G.E. Stedman, M.P. Poulton, C.H. Rowe, Z. Li and P.V. Wells, IEEE Trans. Instrum. Meas. 42 (1993) 407, Erratum 43 (1994) 102.
- [13] G.E. Stedman, M.T. Johnsson, Z. Li, C.H. Rowe and H.R. Bilger, T violation and microhertz resolution in a ring laser (1994) Opt. Lett. to be published.
- [14] G.E. Stedman, Z. Li and H.R. Bilger, Sideband analysis and seismic detection in a precision ring laser (1994) Appl. Opt. submitted for publication.
- [15] G.E. Stedman and H.R. Bilger, Phys. Lett. A 122 (1987) 289.
- [16] H. Primakoff, Phys. Rev. 81 (1951) 899.
- [17] S.L. Adler Ann. Phys. 67 (1971) 599.
- [18] E. Iacopini and E. Zavattini, Phys. Lett. B 85 (1979) 151;
 E. Iacopini, B. Smith, G. Stefanini and E. Zavattini, Nuovo Cim. B 61 (1981) 21;
 G. Cantatore, F. Della Valle, E. Milotti, L. Dabrowski and C. Rizzo, Phys. Lett. B 265 (1991) 418;
 Wei-Tou Ni, Kimio Tsubono, Norikatsu Mio, Kazumichi Narihara, Shen-Che Chen, Sun-Kun King and Sheau-Shi Pan, Mod. Phys. Lett. A 6 (1991) 3671.

Captions to figures:

Fig. 1. Ring-laser polarization geometry for detecting axions and QED vacuum birefringence. Attenuation and retardation of the beam polarized parallel to the external magnetic field modulates the ring laser output signal.

Fig. 2. (a) Photons propagating in a transverse magnetic field producing axions through the Primakoff effect. (b) Virtual axion production leading to vacuum birefringence. (c) QED vacuum birefringence via γ - γ scattering.

Fig. 3. Canterbury ring-laser FM sensitivity. Δf is the peak frequency deviation in Hz. The shaded region indicates ring-laser S/N ratios at present. The dashed line indicates the S/N ratios required to detect QED vacuum birefringence for μHz , mHz and Hz modulation of \vec{B}_{ext} .

Fig. 4. Limits on the axion mass m_a and coupling $g_{\gamma\gamma a}$ to two photons [6]. The heavy dashed line indicates the sensitivity of the proposed ring-laser experiment, conservatively assuming a μHz level frequency resolution. The shaded region indicates the results from the BFR experiment.